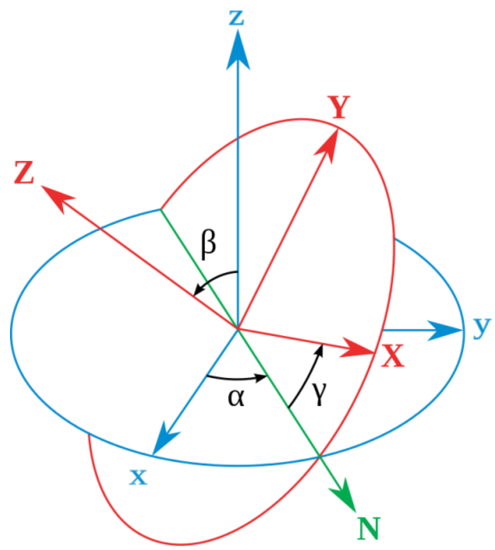
Calypso - Euler Angles



Proper Euler angles geometrical definition. The xyz (fixed) system is shown in blue, the XYZ (rotated) system is shown in red. The line of nodes (N) is shown in green

What Are They?

Euler angles are a series of rotations about three angles, which are shown by *α*, *β*, and *γ*. (Euler angles can also be referred to as *roll*, *pitch*, and *yaw*.) Euler angles describe the orientation with respect to a fixed coordinate system.

Intrinsic Rotations: Rotations around a moving body

Intrinsic rotations are basic rotations that occur about the axes of a coordinate system *XYZ* attached to a moving body. Think robotic arm.

- *x-y-z*, or x_0-y_0-z_0 (initial or fixed coordinate system) shown in blue
- *x'-y'-z'*, or x_1-y_1-z_1 (after first rotation)
- *x″-y″-z″*, or x_2-y_2-z_2 (after second rotation)
- *X-Y-Z*, or x_3-y_3-z_3 (final)

- *α* represents a rotation around the *z* axis,
- *β* represents a rotation around the *x'* axis,
- *γ* represents a rotation around the *z″* axis.

Extrinsic Rotations: Rotations around a fixed coordinate system

Extrinsic rotations are basic rotations that occur about the axes of the fixed coordinate system *xyz*. The *XYZ* system rotates, while *xyz* is fixed. This is how the CMM works using a fixed start point. Starting from the fixed *xyz* coordinate system the *XYZ* is rotated by a series of three rotations to reach any target orientation for *XYZ*. The Euler (*α*, *β*, *γ*) are the results of these basic rotations. For instance, in our next example our orientation can be reached as follows:

- The *XYZ* system rotates about the *z* axis by *α*. The *X* axis is now at angle *α* with respect to the *x* axis of the fixed system.
- The *XYZ* system rotates again about the *x* axis by *β*. The *Z* axis is now at angle β with respect to the *z* axis of the fixed system.
- The *XYZ* system rotates a third time about the *z* axis by *γ*.

In summary, the three basic rotations, in both the intrinsic and extrinsic examples, occur about *z*, *x* and *z*. Indeed, this rotation sequence is often known as *z-x-z* (or 3-1-3).

Euler angles in Calypso are reported in Radians. In our examples below we will multiple by the radian conversion 57.295779513 to convert them back to decimal degrees.

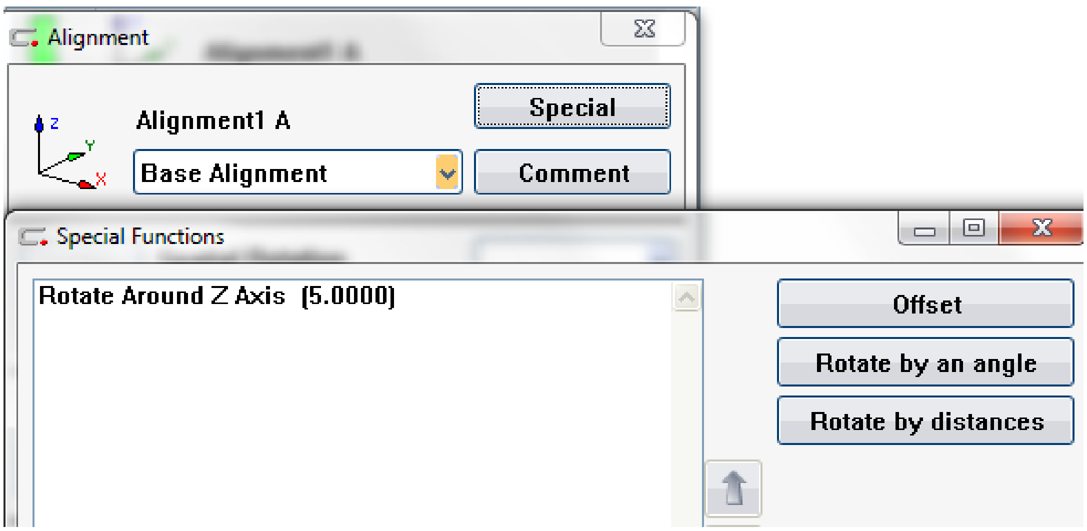
How to Report a Euler Angle

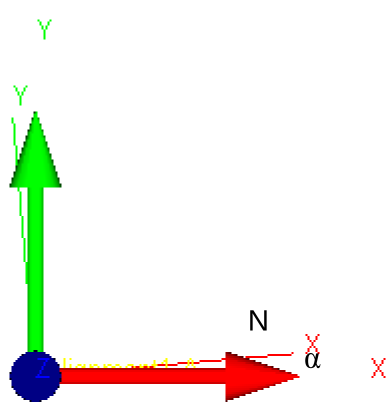
We will use Result Element to report the results of our Euler angle formula.

Let's break this down rotation by rotation for this first example. Our entire rotation sequence in this example will be using the z-x-z or 3-1-3 example as above. Using the rotations below.

About z: 5.000
About x: 40.000
About z: 20.000

First, rotate 5.000° about Z for Alignment1

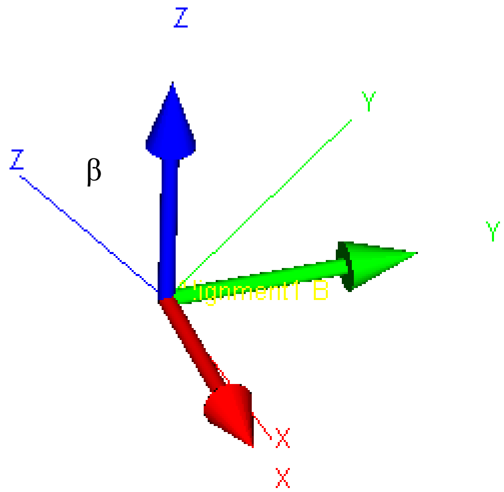




The new X axis becomes our Node (the green line in the above diagram (N)) this gives us the Euler angle α.

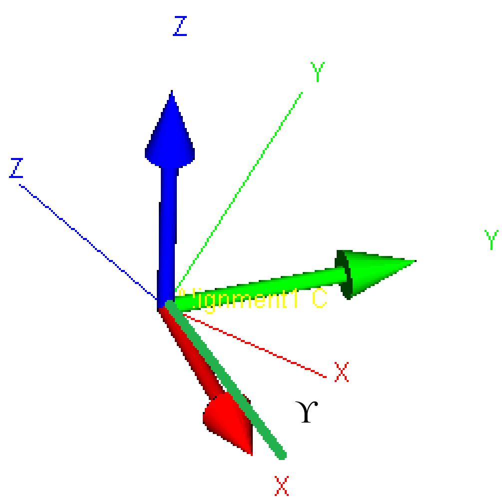
You will see below this is Euler1

Now rotate around X 40.000°



The new Z axis gives us the Euler angle β. In Calypso this will be Euler2.

Now rotate 20.000° About Z. In Calypso this will be Euler4.

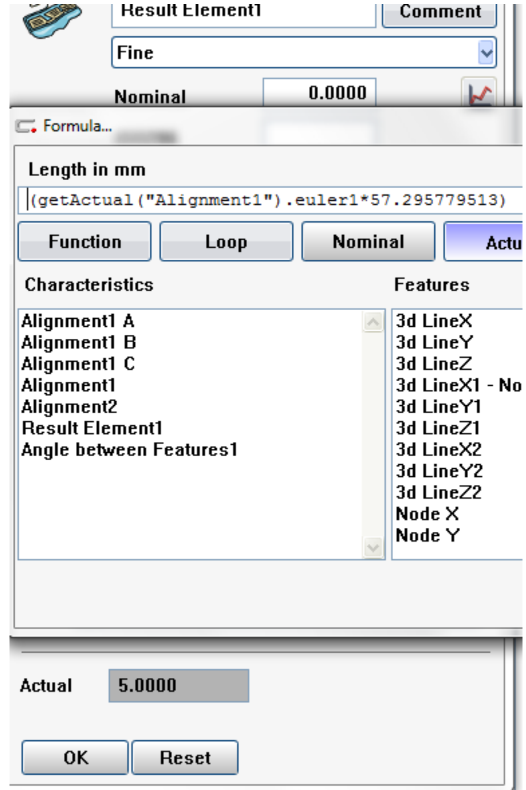


Now let's report the Euler Angles using Result Elements

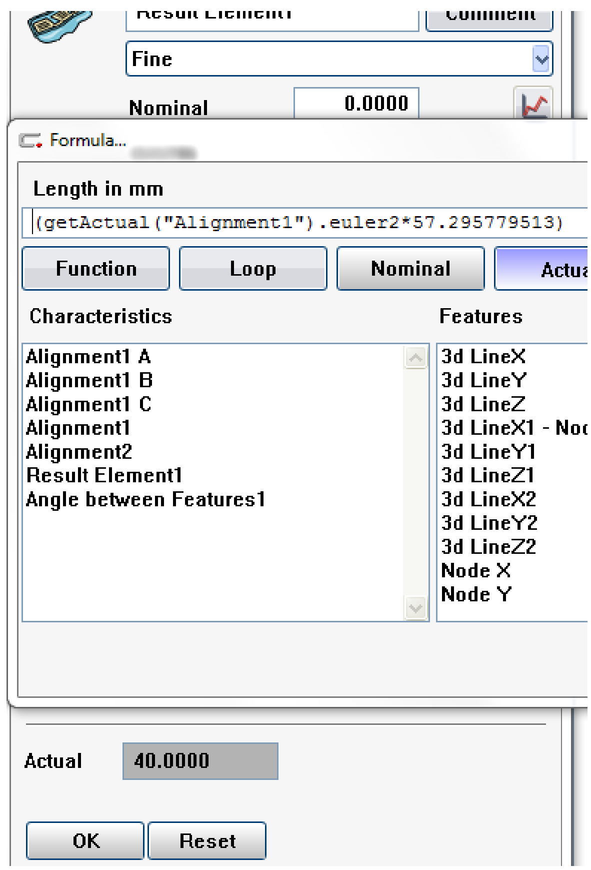
Using the example above we have rotated Alignment1

Using the rotation angles

Euler1 = Y
Euler1 (*57.295779519 to convert from Radian to Decimal Degree) is 5.000°

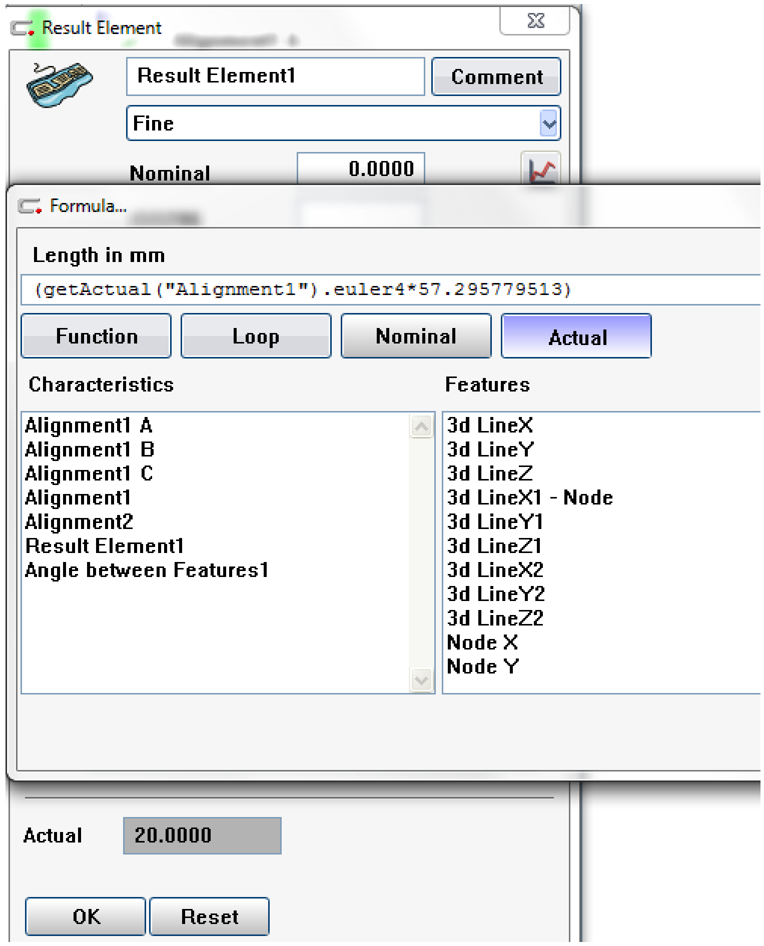


Euler2 = X
Euler2 (*57.295779519 to convert from Radian to Decimal Degree) is 40.000°



Euler4 = X
Euler4 (*57.295779519 to convert from Radian to Decimal Degree) is 20.000°

This is from the Node to the new X created after the final rotation.



In Summary.

α = 5.000° (Euler1)
β = 40.000° (Euler2)
ϒ = 20.000° (Euler4)

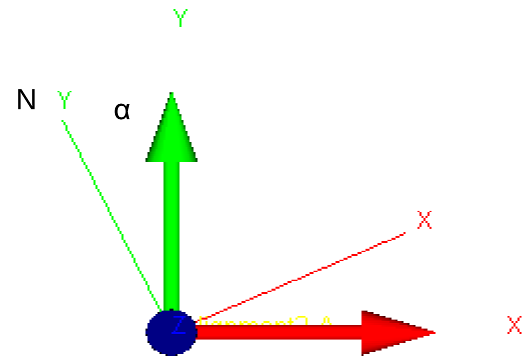
z-y-z or 3-2-3 Rotation

In this example we created an Alignment with the following rotation

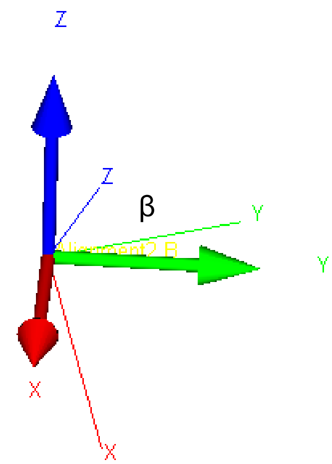
Rotate Around Z Axis (25.0000)
Rotate Around Y Axis (40.0000)
Rotate Around Z Axis (20.0000)

Rotate 25.000° about Z. This creates the new Y or our Node (N)

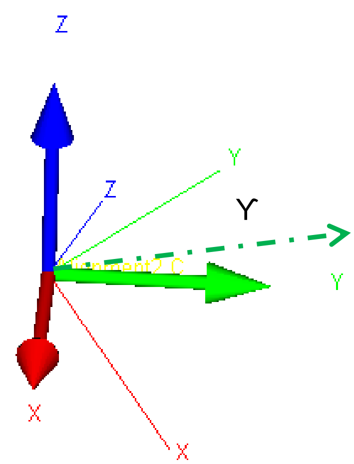
The new Y gives us the Euler angle α.



Rotate 40.000° about Y. This gives us β



Rotate 20.000° about Z. This gives us ϒ
In the diagram the green dashed line is the Node (the new Y location based on the first rotation).



Using Result Element the Euler angles are reported as shown.

Euler 1

115.000° angle (115.00° - 90.00° = 25.000°)

Euler 2

40.000° angle

Euler 4

290.000° angle (290.00° - 270.00° = 20.000°)

Euler angles can be used in formulas, verification of rotational alignments, reporting deviations along a particular axis, etc…. The intent here is to show what they are and how they are reported using Calypso.