

Available online at www.sciencedirect.com

SciVerse ScienceDirect

Physics Procedia

Physics Procedia 24 (2012) 2159 - 2164

# 2012 International Conference on Applied Physics and Industrial Engineering

# Four Methods for Roundness Evaluation

Wentao Sui<sup>1</sup>, Dan Zhang<sup>2</sup>

<sup>1</sup>School of Mechanical Engineering Shandong University of Technology Zibo, China <sup>2</sup>School of Electrical & Electronic Engineering Shandong University of Technology Zibo, China

#### Abstract

Whether roundness error can be evaluated accurately and efficiently or not will directly influence the mechanical products performance and life. For this reason, this paper introduces simple and efficient algorithms to evaluate the roundness error from the measured points using four internationally defined methods: Least Squares Circle (LSC), Minimum Circumscribed Circle (MCC), Maximum Inscribed Circle (MIC) and Minimum Zone Circles (MZC). A software has been developed using Matlab to apply these algorithms on the test data.

© 2011 Published by Elsevier B.V. Selection and/or peer-review under responsibility of ICAPIE Organization Committee. Open access under CC BY-NC-ND license.

Keywords: Roundness error; Optimization; Minimum zone circle; Minimum circumscribed circle; Maximum inscribed circle

# 1. Introduction

Circular feature is one of the most basic geometric elements of mechanical parts [1]. In manufacturing environments, variations on circular features may occur due to imperfect rotation, erratic cutting action, inadequate lubrication, tool wear, defective machine parts, chatter, misalignment of chuck jaws, etc.[2]. Whether roundness error can be evaluated accurately and efficiently or not will directly influence the mechanical products performance and life. Therefore, there is a requirement to develop an automatic inspection method that will satisfy the needs of roundness inspection. Fairly extensive research in the area of roundness evaluation and inspection is still underway [2].

The ANSI dimensioning and tolerance standard Y14.5 defines form tolerances of a component with reference to an ideal geometric feature[3-5]. Various researchers have attempted to develop methods for establishing the reference feature and to evaluate the circularity error. Several geometry measurement techniques are available to estimate the reference feature (circle). These include the Minimum

Circumscribed Circle (MCC), the Maximum Inscribed Circle (MIC), the Minimum Zone Circles (MZC) and the Least Squares Circle (LSC).

This paper introduces simple and efficient algorithms to evaluate the roundness error from the measured points using four internationally defined methods: Least Squares Circle (LSC), Minimum Circumscribed Circle (MCC), Maximum Inscribed Circle (MIC) and Minimum Zone Circles (MZC).

# 2. Methods of evaluating the roundness

### 2.1 Minimum Zone circle (MZC)

In this method, two circles are used as reference for measuring the roundness error. One circle is drawn outside the roundness profile just as to enclose the whole of it and the other circle is drawn inside the roundness profile so that it just inscribes the profile. The roundness error here is the difference between the radius of the two circles. This method is shown in Figure 1.



Figure 1- The Minimum Zone Circle (MZC) method

#### 2.2 Least Squares Circles (LSC)

The least squares circle (LSC) is fitted inside the profile such that the sum of the squares of radial ordinates between the circle and profile is minimized as illustrated in Figure 2. The center of the LSC is then used to draw a circumscribed and an inscribed circle on the polar profile and the out-ofroundness value is the radial separation of these two circles. The least squares circle and its center are unique because there is only one that meets the definition and its accuracy depends on the number of points taken. Manual calculation of the LSC is labored and time consuming but newer digital instruments simplify the process dramatically.



Figure 2-The Least Squares Circle (LSC) method

#### 2.3 Maximum Inscribed Circle (MIC)

This method fits the largest possible circle inside the profile figure as shown in Figure 3. The circle can be determined by trial and error with a compass or with a template. After the circle has been drawn, the out-ofroundness value is the maximum distance between the profile and the inscribed circle.



Figure 3 - The Maximum Inscribed (MIC) method

## 2.4 Minimum Circumscribed Circle (MCC)

A center is found by drawing a circle that has the smallest possible radius but still contains the polar plot profile in this method as illustrated in Figure 4. An inscribed circle is then drawn inside the profile based on the center of the minimum circumscribed circle. The out-of-roundness value is the difference between the radii of the inscribed and circumscribed circle.



Figure 4- The Maximum Circumscribed Circle (MCC) method

# 3. The mathematic models for reference circle

The roundness error can be evaluated through solving nonlinear unconstrained optimization problems. As long as the mathematical objective function is build correctly, the results can be obtained without the preparation of the solution process, therefore, the solution process is very simple.

# 3.1 Minimum Zone circle (MZC)

The MZC method uses the minimum zone circle as the reference circle to evaluate the roundness. The roundness error is expressed as follows:

 $\Delta Z_{z} = R_{\text{max}} - R_{\text{min}} \qquad (1)$ 

where  $R_{\text{max}}$  and  $R_{\text{min}}$  is the maximum and minimum distance between the MZC circle and the measured profile.

Suppose a round in the XOY plane or in parallel to the plane XOY, the standard equation of the circle is

$$(x - x_c)^2 + (y - y_c)^2 = R_c^2 \qquad (2)$$

where  $x_c$  and  $y_c$  is the center coordinates of the circle. For each measured point  $Mi(x_i, y_i)$ , the distance between every points and the circle center equals:

$$R_{i} = (x_{i} - x_{c})^{2} + (y_{i} - y_{c})^{2} \quad (3)$$

The objective function is minimizing the distance between two concentric circle tangent exterior and interior to real profile. The function has following form:

$$F(x_{c}, y_{c}) = Min\{[R_{i}]_{\max} - [R_{i}]_{\min}\}$$
(4)

where  $[R_i]_{\text{max}}$  represents the radius of outer tangent circle to measured points set and  $[R_i]_{\text{min}}$  represents the radius of inner tangent circle to measured points set. Developing the relation (4) using relation (3) is resulting:

$$F(x_{c}, y_{c}) = Min\{Max(\sqrt{(x_{i} - x_{c})^{2} + (y_{i} - y_{c})^{2}}) (5) - Min(\sqrt{(x_{i} - x_{c})^{2} + (y_{i} - y_{c})^{2}})\}$$

#### 3.2 Least Squares Circles

The objective function is minimizing the sum of square deviations from measurement points to radius of substitute circle (see Figure 2). The objective function has following form:

$$F(x_{c}, y_{c}, R_{c}) = Min(\sum_{i=1}^{n} R_{i}^{2})$$
(6)

# 3.3 Maximum Inscribed Circle

The objective function is maximizing the radius of circle tangent interior to real profile (see Figure 3). The function is as follow:

$$F(x_c, y_c) = Max \{Min[R_i]\}$$
(7)

# 3.4 Minimum Circumscribed Circle

The objective function is minimizing the radius of circle tangent exterior to real profile (see Figure 4). The relation of objective function is as follow:

$$F(x_c, y_c) = Min \{Max [R_i]\}$$
(8)

## 4. Experiment Demonstration

For the purpose of testing the validity of the above methods, 20 test data are selected which is shown in Table 1. The results are tabulated in Table 2.

Table 1 The coordinates of measured points and intermediate process of four methods

	V ()	V ()	MZC	LSC	MCC	MIC
	A (IIIII)	Y (IIIII)	(um)	(um)	(um)	(um)
1	-15.4512	47.5540	-0.1	1.1	-0.4	2
2	-29.3892	40.4508	-1.6	-0.4	-2	0.5
3	-40.4511	29.3895	-1.4	-0.1	-1.9	0.8
4	-47.553	15.4509	-1.7	-0.3	-2.3	0.5
5	-49.9998	0.0000	-2.2	-0.8	-2.8	0.2
6	-47.5535	-15.4511	-1.4	0.1	-1.8	1.2
7	-40.4504	-29.3889	-2.7	-1.1	-3	0
8	-29.3905	-40.4526	0	1.7	0	2.9
9	-15.4508	-47.5527	-2.2	-0.5	-1.9	0.8
10	0.0000	-50.0001	-1.8	-0.1	-1.3	1.3
11	15.4512	-47.5538	-0.8	1	0	2.5
12	29.3893	-40.4509	-1.5	0.1	-0.6	1.7
13	40.4508	-29.3892	-1.5	0.1	-0.5	1.7

14	47.5520	-15.4506	-2.1	-0.6	-1	0.9
15	49.9996	0.0000	-1.5	0	-0.5	1.4
16	47.5512	15.4503	-2.7	-1.4	-1.8	0
17	40.451	29.3893	-0.8	0.5	-0.1	1.7
18	29.3894	40.4511	-0.7	0.5	-0.2	1.7
19	15.4505	47.5517	-2.3	-1	-2	0
20	0.0000	50.0012	0	1.2	0	2.2

Table 2 The result obtained from four methods

	MZC	LSC	MCC	MIC
Roundness error	2.7	3.1	3.0	2.9
maximum deviation	0.0	1.7	0.0	2.9
minimum deviation	-2.7	-1.4	-3.0	0.0

The equation of reference circle using the MZC method is

$$(x+0.0005)^{2} + (y+0.0004)^{2} = 50.00162^{2}$$
 (9)

The equation of reference circle using the LSC method is:

$$(x+0.0005)^{2} + (y+0.0001)^{2} = 50.00012^{2}$$
(10)

The equation of reference circle using the MCC method is:

$$(x+0.0012)^{2} + (y+0.0001)^{2} = 50.00132^{2} \quad (11)$$

The equation of reference circle using the MIC method is:

$$(x+0.0007)^{2} + (y+0.0001)^{2} = 49.9989^{2}$$
(12)

#### 5. Discussion and conclusion

This paper introduces simple and efficient algorithms to evaluate the roundness error from the measured points using four methods: LSC, MCC, MIC and MZC. Among the four methods, only the MZC complies with ISO standards and has the minimum roundness error value. The LSC method is robust, but it does not guarantee the minimum zone solution specified in the standards.

#### References

[1]X.M. Li., Z.Y Shi, " The relationship between the minimum zone circle and the maximum inscribed circle and the minimum circumscribed circle," Precision Engineering, vol. 33, No.3, pp. 284-290.

[2]E.S. Gadelmawla, "Simple and efficient algorithms for roundness evaluation from the coordinate measurement data," Measurement, Vol. 43, No.2, pp. 223-235, February 2010

[3] H. Chang, T.V. Lim, "Evaluation of circularity tolerance using Monte Carlo simulation for coordinate measuring machine,"International Journal of Production Research 31 (1993) 2079–2086.

[4] C. Mu-Chen, T. Du-Ming, T. Hsien-Yu, , "A stochastic optimization approach for roundness measurements,"Pattern Recognition Letters 20 (1999) 707-719

[5] ANSI/ASME Y14.5M, Dimensioning and Tolerancing, American Society of Mechanical Engineers, New York, 1982.